**Appendix A: Descriptions of the models used in the Monte Carlo simulations, for each of the three models: CUYA, VERM, and MAUM.**

**CUYA model (based on Cuyahoga River at Independence, OH, chloride data for water years 2004-2013).**

The model of discharge as a function of time of year can be represented by figure 1.



Figure . Graph showing the median and 5th and 95th percentiles of the distribution of ln(Q) by time of year, for the CUYA model.

The selected ARMA model for discharge is ARMA(2,2): the AR coefficients are 1.1452 and – 0.2256 respectively. The MA coefficients are -0.1863 and -0.2206 respectively. The theoretical autocorrelation function (ACF) for this model is shown in figure 2.



Figure . Graph showing the autocorrelation function (ACF) for lags of 1 day through 60 days for deseasonalized ln(Q) values in the CUYA model.

The selected year for the water quality model is 2008 and Figure 3 shows the surface that represents the function (the mean of ln(c)).



Figure . Contour plot showing the mean of ln(c) as a function of time of year and discharge, used in the CUYA model. The relationship repeats over all years in each of the Monte Carlo simulations.



Figure . Contour plot showing the standard deviation of ln(c) as a function of time of year and discharge, for the CUYA model. This pattern repeats over all of the years in each of the Monte Carlo simulations.

The selected ARMA model for the standardized residuals from the model represented by figures 3 and 4 is an ARMA(2,1) model: the AR coefficients are 1.5915 and -0.6094 respectively, the MA coefficient is -0.8585. The theoretical autocorrelation function for this ARMA(2,1) model is depicted in figure 5.



Figure . The autocorrelation function (ACF) for the ARMA(2,1) used to simulate the standardized residuals simulated in the CUYA model.

**VERM model (based on data from Vermilion River, at Pontiac, IL for from 1992-10-01 through 1999-04-30 for dissolved nitrate as mg/L N).**

The model of discharge as a function of time of year can be represented by figure 6.



Figure 6: The median and 5th and 95th percentiles of the distrbution of ln(Q) by time of year, for the VERM model.

The selected ARMA model for discharge is an ARMA(3,3). The AR coefficents are -0.4798, 0.883, and 0.4215 respectively. The MA coefficients are 2.0028, 1.3586, and 0.3411 respectively. The autocorrelation function for this ARMA(3,3) model is shown in figure 7.



Figure 7: The autocorrelation function (ACF) for lags of 1 day through 60 days for deseasonalized ln(Q) values in the VERM model.

The selected year for the water quality model is 1995 and figure 8 shows the surface that represents the function (the mean of ln(c)).



Figure 8: Contour plot showing the mean of ln(c) as a function of time of year and discharge, used in the VERM model. The relationship repeats over all years in each of the Monte Carlo simulations.

Figure 9 shows the surface that represents the function (the standard deviation of ln(c)).



Figure 9. Contour plot showing the standard deviation of ln(c) as a function of time of year and discharge for the VERM model. This pattern repeats over all of the years in each of the Monte Carlo simulations.

The selected ARMA model for the standardized residuals from the model represented by figures 8 and 9 is an ARMA(4,4) model: the AR coefficients are 0.136, 1.0266, 0.5265, and -0.7044, respectively, and the MA coefficients are 0.436, -0.6653, -0.8115, and 0.2004, respectively. The theoretical ACF for this model is shown in figure 10.



Figure 10. The autocorrelation function (ACF) for the ARMA(4,4) model used to simulate the standardized residuals in the VERM model.

**MAUM model (based on Maumee River at Watertown, OH, total phosphorus data for water years 2006-2013)**

The model of discharge as a function of time of year can be represented by Figure 11.



Figure 11. The median and 5th and 95th percentiles of the distribution of ln(Q) by time of year, for the MAUM model.

The selected ARMA model for discharge is an ARMA(2,1) model: the AR coefficients are 1.1154 and -0.1975 respectively, the MA coefficient is 0.2695. The theoretical ACF for this model is shown in Figure 12.



Figure 12. The autocorrelation function (ACF) for lags of 1 day through 60 days for deseasonalized ln(Q) values in the MAUM model.

The selected year for the water quality model is 2009 and Figure 13 shows the surface that represents the function (the mean of ln(c))



Figure 13. Contour plot showing the mean of ln(c) as a function of time of year and discharge, used in the MAUM model. The relationship repeats over all years in each of the Monte Carlo simulations.

Figure 14 shows the surface that represents the function (the standard deviation of ln(c)).



Figure 14. Contour plot showing the standard deviation of ln(c) as a function of time of year and discharge, for the MAUM model. This pattern repeats over all of the years in each of the Monte Carlo simulations.

The selected ARMA model for the standardized residuals from the model represented by figures 13 and 14 is an ARMA(2,1) model: the AR coefficients are 1.1495 and -0.2276 respectively, the MA coefficient is -0.5307. The theoretical ACF for this model is shown in Figure 15.



Figure 15. The autocorrelation function (ACF) for the ARMA(2,1) used to simulate the standardized residuals in the MAUM model.